Engineering Mathematics I

Unit II

Fourier Series

1 A function $f(x)$ is said to be periodic of period T if

a) $f(x+T)=f(x),\ \forall x$
b) $f(x+T)=f(T),\ \forall x$
c) $f(-x)=f(x),\quad \forall x$
d) $f(-x)=-f(x),\quad \forall x$

2 Fourier series representation of periodic function $f(x)$ with period 2π which satisfies the Dirichlet's conditions is

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$
b) $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos n\pi x+b_n\sin n\pi x)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx)(\,b_n\sin nx)$
d) $\frac{a_0}{2}+(a_n\cos nx+b_n\sin nx)$

3 The Fourier series of an odd periodic function contains only

a) Odd harmonic
b) Even harmonic
c) Cosine terms
d) Sine terms

4 The trigonometric series of an even function does not have

a) Constant
b) Sine terms
c) Cosine terms
d) Odd harmonic terms

5 If $f(x+nT)=f(x)$ where n is any integer then the fundamental period of $f(x)$ is

a) $2T$
b) $T/2$
c) T
d) 3T

6 If $f(x)$ is a periodic function with period T then $f(ax), a\neq 0$ is periodic function with fundamental period

a) T
b) T/a
c) aT
d) π

7 If $f(x)=-f(-x)$ and $f(x)$ satisfy the Dirichlet's conditions, then $f(x)$ can be expanded in a Fourier series containing

a) Only sine terms
b) Cosine terms and constant term
c) Only cosine terms
d) Sine terms and constant term

8 Fundamental period of $\cos 2x$ is

a) $\frac{\pi}{4}$
b) $\frac{\pi}{2}$
c) π
d) 2π

9 Fundamental period of $\tan 3x$ is

a) $\frac{\pi}{2}$
b) $\frac{\pi}{3}$
c) π
d) $\pi/4$

10 The value of constant terms in the Fourier series of $f(x)=e^{-x}$ in $0\leq x\leq 2\pi$, $f(x+2\pi)=f(x)$ is

a) $\frac{1}{\pi}(1-e^{-2\pi})$
b) $\frac{1}{2\pi}(1-e^{-2\pi})$
c) $2(1-e^{-2\pi})$
d) $(1-e^{-2\pi})$

11 If $\psi(x)=f(x)-f(-x)$ and $\psi(x)$ satisfy the Dirichlet's conditions, then $\psi(x)$ can be expanded in a Fourier series containing

a) Only sine terms
b) Cosine terms and constant term
c) Only cosine terms
d) Sine terms and constant term

12 If $\psi(x)=f(x)+f(-x)$ and $\psi(x)$ satisfy the Dirichlet's conditions, then $\psi(x)$ can be expanded in a Fourier series containing

a) Only sine terms
b) Cosine terms and constant term
c) Only cosine terms
d) Sine terms and constant term

13 If $f(x)$ is periodic function with period $2L$ defined in the interval C to $C+2L$ then Fourier coefficient a_0 is

a) $\int_{C}^{C+2L} f(x)\,dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$

d) $\frac{1}{L}\int_{C}^{C+2L} f(x)\;dx$

14 If $f(x)$ is periodic function with period $2L$ defined in the interval C to $C+2L$ then Fourier coefficient a_n is

a) $\int_{C}^{C+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$

d) $\frac{1}{L}\int_{C}^{C+2L} f(x)\;dx$

15 If $f(x)$ is periodic function with period $2L$ defined in the interval C to $C+2L$ then Fourier coefficient b_n is

a) $\int_{C}^{C+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$

d) $\frac{1}{L}\int_{C}^{C+2L} f(x)\;dx$

16 For an even function $f(x)$ defined in the interval $-\pi \le x \le \pi$ and $f(x+2\pi)=f(x)$ the Fourier series is

a) $\sum_{n=1}^{\infty} b_n \sin nx$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

d) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

17 For an odd function $f(x)$ defined in the interval $-\pi \le x \le \pi$ and $f(x+2\pi)=f(x)$ the Fourier series is

a) $\sum_{n=1}^{\infty} b_n \sin nx$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

d) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

18 Fourier coefficient for an odd function $f(x)$ defined in the interval $-L \le x \le L$ and $f(x+2L) = f(x)$ are

a) $a_0 = 0, a_n = 0, b_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

b) $a_0 = \frac{2}{L}\int_0^L f(x)\,dx,$ $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx, b_n = 0$

c) $a_0 = 0, a_n = 0, b_n = 0$

d) $a_0 = 0, a_n = 0,$ $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

19 Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

a) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

b) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

c) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

d) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

20 Half range Fourier sine series for $f(x)$ defined in the interval $0 \le x \le L$ is

a) $\sum_{n=1}^{\infty} b_n \sin\frac{nx}{L}$

b) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

c) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

d) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

21 In Harmonic analysis the term $a_2 \cos 2x + b_2 \sin 2x$ is called

a) Second harmonic
b) First harmonic
c) Third harmonic
d) None of these

22 In Harmonic analysis the amplitude of first harmonic $a_1 \cos x + b_1 \sin x$ is

a) $\sqrt{a_1^2 - b_1^2}$
b) $a_1^2 + b_1^2$
c) $\sqrt{a_1^2 + b_1^2}$
d) $(a_1^2 + b_1^2)^2$

23 For the certain data if $a_0 = 1.5,\ a_1 = 0.373,\ b_1 = 1.004$ then the amplitude of 1st harmonic is

a) 1.07
b) 2.07
c) 1.004
d) 1.377

24 Fourier series representation of periodic function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

$f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$ then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$

a) $\frac{\pi^2}{4}$
b) $\frac{\pi^2}{8}$
c) $\frac{\pi^2}{16}$
d) $\frac{8}{\pi^2}$

25 Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$ then value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots =$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$

c) $\frac{\pi^2}{6}$ d) $\frac{\pi^2}{12}$

26 Fourier coefficient a_0 in the Fourier series expansion of $f(x) = e^{-x}; 0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is

a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{2\pi})$

c) $\frac{2}{\pi}(e^{-2\pi} - 1)$ d) $\frac{1}{\pi}(1 + e^{2\pi})$

27 Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2; 0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{6}$

c) 0 d) $\pi/6$

28 Fourier coefficient a_0 in the Fourier series expansion of $f(x) = x\sin x; 0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$

a) 2 b) 0

c) -2 d) -4

29 $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$ Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$, then Fourier coefficient a_0 is

a) 2π b) $\pi/3$

c) 0 d) $\pi/2$

30 The Fourier constant a_n for $f(x) = 4 - x^2$ in the interval $0 < x < 2$ is

a) $4/\pi^2 n^2$ b) $2/n^2\pi^2$

c) $4/n^2\pi$ d) $2/n\pi^2$

31 $f(x) = x, -\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$, Fourier coefficient b_1 is

a) 2 b) -1

c) 0 d) $2/\pi$

32 For half range sine series of $f(x) = x, 0 \le x \le 2$ and period is 4.Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$, then Fourier coefficient b_1 is

a) 4 b) 2

c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$

33 Fourier series representation of periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$ is $\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$, then value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots =$

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$

c) $\pi^2/3$ d) 0

34 The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

a) 17.85 b) 20.83
c) 35.71 d) 41.66

35 The value of a_0 in harmonic analysis of y for the following tabulated data is

x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

a) 1.45 b) 5.8
c) 2.9 d) 2.48

36 The value of b_1 in Harmonic analysis of y for the following tabulated data is :

x°	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17.3	11.7	0
$\sin 2x$	0	0.866	0.866	0	-0.866	-0.866	0

a) -3.116 b) -1.558
c) -4.16 d) -1.336

37 The value of a_1 in Harmonic analysis of y for the following tabulated data is :

x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1

a) -4.16 b) -8.32
c) -3.57 d) -10.98

38 The values of a_1, b_1 in Harmonic analysis of y for the following tabulated data with period 2π are respectively :

x	0	$\pi/2$	π	$3\pi/2$
y	1	2	3	2

a) $-2, 2$ b) $0, 2$
c) $2, -2$ d) $-2, 0$

39 The value of a_2 in Harmonic analysis of y for the following tabulated data with period 2π is

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
y	0	1	3	5	7	6	4	2

a) -2 b) 0
c) 2 d) $-1/4$

40 The value of a_1, a_2 in Fourier cosine series of y for the following tabulated data are

x	0	$\pi/4$	$\pi/2$	$3\pi/4$
y	0	$\sqrt{2}$	2	$\sqrt{2}$

a) $-1/2, 1/2$ b) $-1/2, -1/2$
c) $2, -2$ d) $-2, 0$

Fourier Series									
01 -a)	02 -b)	03 - d)	04 - b)	05 - c)	06 - b)	07 - a)	08 - c)	09 - b)	10 - b)
11 - d)	12 - b)	13 - d)	14 - c)	15 - b)	16 - c)	17 - a)	18 - d)	19 - c)	20 - b)
21 - a)	22 -c)	23 - a)	24 - b)	25 - d)	26 - a)	27 - b)	28 - c)	29 - d)	30 - a)
31 - a)	32 - d)	33 - a)	34 - d)	35 - c)	36 - b)	37 - d)	38 - d)	39 - b)	40 - b)